Optimal Shaping of the Integral Bridge Girder by Means of Optimal Control

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Abstract. This paper investigates the optimal shaping of the bottom flange height of an I-section steel beam constituting part of an integral steel- concrete bridge girder. The problem is formulated as a control theory task. Consecutive work phases (construction and exploitation) with different static schemes and loadings are incorporated in the task formulation. The solution is compared to results obtained from the finite element software Abaqus.

INTRODUCTION

The Pontryagin's maximum principle is applied in this article to the shape optimization of the integral steel - concrete bridge girder. The optimization concerns the search for the decision variable: the height of the bottom flange of the I section steel beam, which minimizes the mass of the steel girder, while meeting the design conditions and restrictions imposed by standards. The mathematical model of the analyzed problem is described by a set of differential state equations, with restricting conditions limiting the maximal stresses and deflections in the integral girder under combinations of the appropriate loads. The multipoint boundary value problem is solved numerically.

TASK FORMULATION WITHIN THE FORMAL STRUCTURE OF THE MAXIMUM PRINCIPLE

The formal structure of the analyzed optimization task consist of: state equations with boundary conditions, internal point conditions, limits on maximal stresses and deflections, and the objective function. This structure must be formulated separately for every structure work phase defined by the static scheme and the load configuration. Two design situations are considered in the analyzed problem:

- Construction phase –the girder works as a continuous, two span steel beam with an additional construction support in the mid-span, loaded by the dead load, and fresh concrete with the formwork.,
- Exploitation phase – the structure works as an integral steel-concrete simply supported girder, loaded with the permanent loads, the crowd loading, and the single axle moving load, according to Eurocode standard [1]. Six different positions of the moving load are taken into account in the optimization problem.

Finally, the set of 37 differential state equations with boundary and internal point conditions describing the optimization problem is formulated. The structure of this optimization allows for a simultaneous introduction of all load states and static schemes into a mathematical model, which is crucial because of numerous and complex design conditions incorporated in the task.

NUMERICAL ANALYSIS

Application of the maximum principle rules to the optimization task brings it to a multipoint boundary value problem, which is solved with the use of the Dircol-2.1 computer software [2]. The obtained solution, namely the trajectory of the height of the I section bottom flange, fulfilling all necessary optimization conditions and assumed restrictions, is depicted in Fig. 1.

Hence only the six chosen positions of the moving load are taken into account during the task formulation, the obtained optimal girder is analysed in the FEM code Abaqus under the action of all loads imposed by standards.. The envelopes of normal stresses and deflections, presented in Fig. 2, confirm that the resultant optimal girder meets all predetermined design conditions. The benefit arising from optimization constitutes the 22% reduction in the steel volume, compared to the initial beam shape with the bottom flange height equal to 0.04m.

The results of the performed analysis confirm that the theory of optimal control in combination with the FEM computations can be successfully applied to structure optimal shaping. The optimal solutions can be used in practical applications or, at a minimum, be a measure of correct design.

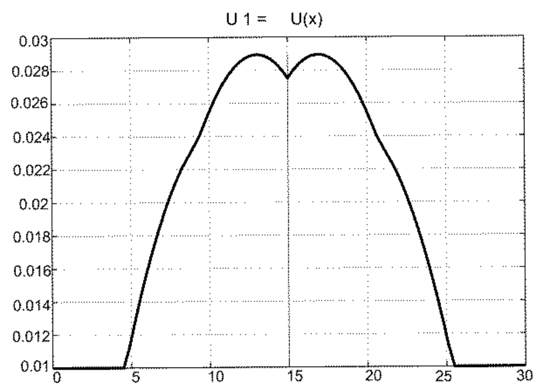

Figure 1. Control variable- the optimal height of the steel beam I section bottom flange [m]

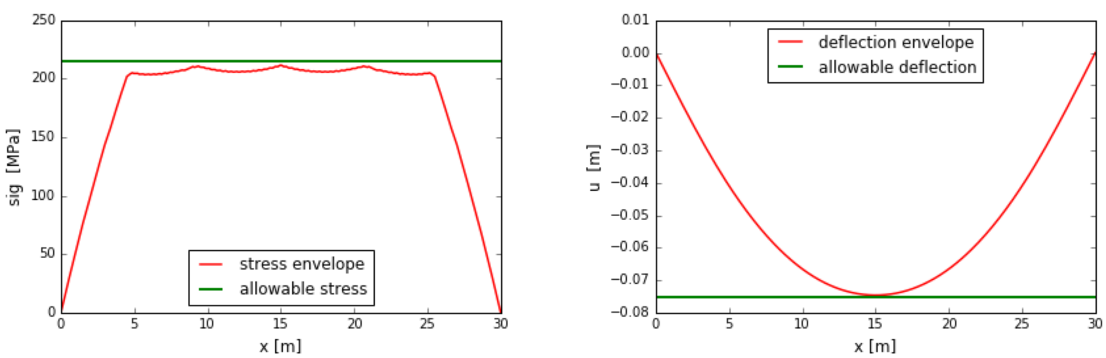

Figure 2. Stress and deflection envelopes in the optimal girder

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